MARLEY

RADIATIVE TRANSFER LECTURE III

EQUATION OF RADIATIVE TRANSFER

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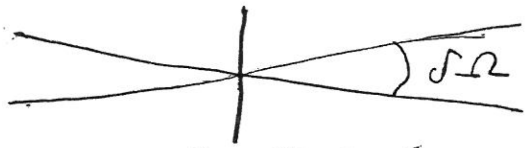
? M B f

DEFINITIONS

SPECIFIC INTENSITY — I (energy/time/area/ster

Energy Flux/solid ∡ crossing a surface of unit area ~~that~~ is aligned ⊥ to the direction of propogation of the radiation



$$\delta F = I\,\delta\Omega\,\delta A = N\,\underbrace{(h\nu)}\,c\,\delta\Omega\,\delta A$$

δF: energy flux

N: # photons/~~ster~~ unit volume

$(h\nu)$: energy/photon

c: speed of light

NET FLUX — F_{net} (energy/time/area)

Radiation energy flux crossing a surface of unit area in ALL Directions

$$F_{net} = \int N(h\nu)(\vec{c}\cdot\hat{n})\,d\Omega$$

$$= \int I\mu\,d\Omega \qquad (\mu = \cos(\theta))$$

$$= \int_{-1}^{1}\int_{0}^{2\pi} I\mu\,d\phi\,d\mu \qquad \left(\text{if } I \neq f(\theta) \quad F = 0\right)$$

spherical coordinates — ϕ = azemith ∡ (≈ longitude)

θ (≈ colatitude, = 0 in $\hat{n}$ direction)

HEMISPHERICAL FLUXES

UPWARD, $F^{+} = \int_0^1 \int_0^{2\pi} I \mu \, d\varphi \, d\mu$

DOWNWARD, $F^{-} = -\int_{-1}^0 \int_0^{2\pi} I \mu \, d\varphi \, d\mu$

MEAN INTENSITY, J

$$J = \frac{1}{4\pi} \int_{-1}^1 \int_0^{2\pi} I \, d\varphi \, d\mu$$

$$J = \frac{Uc}{4\pi}$$

U = ENERGY DENSITY OF RADIATION FIELD

$= N(h\nu) 4\pi$

RADIATION PRESSURE, P_R

mom. flux

$$P_R = \frac{1}{c} \int_{-1}^1 \int_0^{2\pi} I \mu^2 \, d\varphi \, d\mu$$

notes:

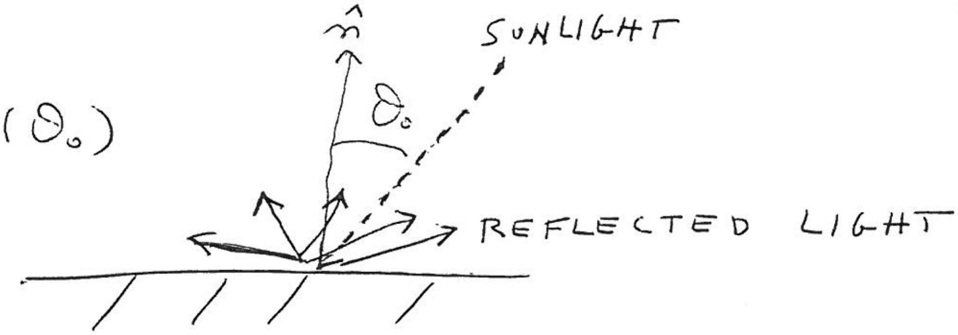
Each photon has momentum $\frac{h\nu}{c}$

momentum flux/ster $\sim \frac{I}{c} \mu \, d\Omega$

normal component $\sim \frac{I}{c} \mu^2 d\Omega$

LAMBERT SURFACE

Simple E balance



$\mu_0 = \cos(\theta_0)$

UNIDIRECTIONAL SUNLIGHT INCIDENT ON LAMBERT SURFACE (EQUALLY BRIGHT AT ALL ANGLES OF REFLECTION):

UPWARD INTENSITY $= I_L =$ CONSTANT

LET λ_L BE THE FRACTION OF SUNLIGHT THAT IS ~~REFLECTED~~ DIFFUSELY REFLECTED (REST IS ABSORBED)

$$F^+ = \lambda_L F^-$$

up flux, down flux

λ = reflectivity coef

BUT

$$F^+ = \int_0^1 \int_0^{2\pi} I_L \mu \, d\phi \, d\mu = \pi I_L$$

$$F^- = \int_0^{-1} \int_0^{2\pi} \mathcal{F}_\odot \, \delta(\mu+\mu_0)\, \delta(\phi-\phi_0)\, \mu \, d\phi \, d\mu$$

Dirac Delta Function

$$= \mu_0 \mathcal{F}_\odot$$

solar flux across surface $\perp$ its direction of propogation

$\longrightarrow$

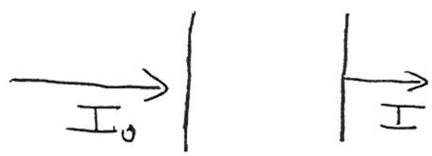
$$I_L = \lambda \mu_0 \frac{\mathcal{F}_\odot}{\pi} \equiv \lambda \mu_0 F_\odot$$

FIDUCIAL INTENSITY UNITS – I/F

$I/F = 1$ FOR A 100% REFLECTIVE ($\lambda = 1$) LAMBERT SURFACE THAT IS NORMALLY ILLUMINATED ($\mu_0 = 1 \rightarrow \theta_0 = 0°$)

OPTICAL DEPTH – τ (dimensionless)

measures exponential attenuation of light passing thru an absorbing/scattering medium



$$I = I_0 \exp(-\tau)$$

Not total I that see, need to account for scattering

Notes:

1. The scattered part of the light is NOT lost. It reappears at a variety of directions of propogation

2. τ is found from the extinction coefficient/length, k_{ext}

$$\tau = \int k_{ext}\, ds$$

ds – path length element

$$k_{ext} = N_p \sigma_p^{ext} + N_g \sigma_g^{ext}$$

N_p – # particles/volume

σ_p^{ext} – extinction X section per particle

N_g – # gas molecules/vol.

σ_g^{ext} – extinction X section per molecule

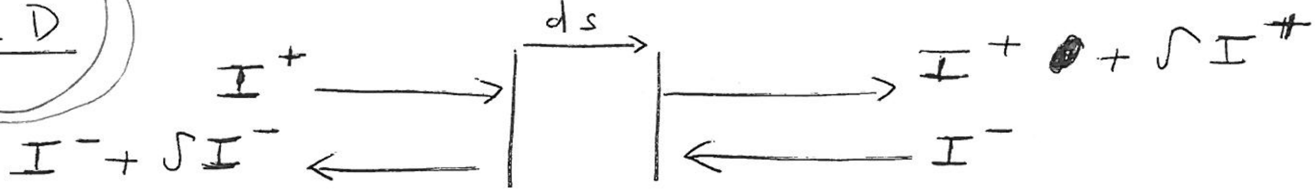
$$\sigma_g^{ext} = \sigma_g^{scat} + \sigma_g^{abs}$$

σ_g^{scat} – Rayleigh scattering

σ_g^{abs} – absorption

EQUATION OF RADIATIVE TRANSFER

1 D



$$\delta I^+ = \text{sources} - \text{sinks}$$

$$\text{sinks} = k_{ext}\, ds\, I^+$$

$$\text{sources} = \text{scattered light} + \text{emitted light}$$

$$\text{scattered light} = k_{scat}\, ds\, (I^+ p^+ + I^- p^-)$$

$$p^+ + p^- = 1$$

fraction of scattered light that is scattered that goes in I^+ direction

forward ~~[illegible]~~ backward scattered fractions

$$\text{emitted} = k_{em}\, ds\, B$$

$\approx$ Black body function

$$\longrightarrow \quad \delta I^+ = k_{scat}\, ds\, (I^+ p^+ + I^- p^-) + k_{em}\, ds\, B - k_{ext}\, ds\, I^+$$

τ positive

Let $d\tau = -k_{ext}\, ds$ (Note $-$ sign)

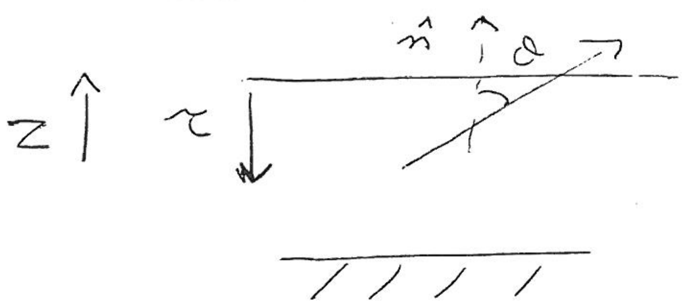
$$\longrightarrow \quad \frac{dI^+}{d\tau} = I^+ - \tilde{\omega}_0 (I^+ p^+ + I^- p^-) - (1 - \tilde{\omega}_0) B$$

Similarly

$$\frac{dI^-}{d\tau} = -I^- + \tilde{\omega}_0 (I^+ p^+ + I^- p^-) + (1 - \tilde{\omega}_0) B$$

true only @ discrete wavelength

PLANE PARALLEL ATMOSPHERE



I, k, B are functions only of altitude

($\neq$ horizontal position)

Good Approximation for
<u>thin atmospheres</u>: $h << R$

height of atmosphere (or scale height)

planet's radius

<u>not</u>
comets
red giants

<u>NOT TOO CLOSE TO horizon</u>

$$|\Theta| < 90^\circ - \epsilon_\theta$$

$$(\mu > \epsilon_\mu \sim .1 - .2)$$

from $ds = \frac{dz}{\mu}$

$$\mu \frac{dI}{d\tau} = \underset{\uparrow \text{ extinction}}{I} - \underset{\uparrow \text{ sources}}{S}$$

$$S = \tilde{\omega}_0 \int_{-1}^{1}\int_0^{2\pi} I \, p(\Theta) \frac{d\varphi \, d\mu}{4\pi}$$

Θ: ∡ scatter $\neq \mu$

$$+ (1 - \tilde{\omega}_0) B$$

B: planck Function

~~by law of cosines:~~ = scattered light +

~~cos(Θ)~~ thermal emission

<u>FORMAL</u> <u>SOLUTION</u> OF THE EQUATION OF RT:

$$I^+(\tau) = S^+(\tau_s)\, e^{-(\tau_s - \tau)/\mu} + \int_{\tau}^{\tau_s} S^+(\tau')\, e^{-(\tau'-\tau)/\mu}\, \frac{d\tau'}{\mu}$$

upward directed intensity

$$I^-(\tau) = S^-(\tau=0)\, e^{-\tau/\mu} + \int_0^{\tau} S^-(\tau')\, e^{-(\tau-\tau')/\mu}\, \frac{d\tau'}{\mu}$$

downward directed intensity

<u>EXAMPLES OF APPLICATIONS:</u>

- THERMALLY EMITTING, NON-SCATTERING ATMOSPHERE + SURFACE $\rightarrow$ $S = B$
  exactly true
- OPTICALLY THIN ATMOSPHERE ILLUMINATED BY SUNLIGHT

<u>GENERALLY HOWEVER</u>

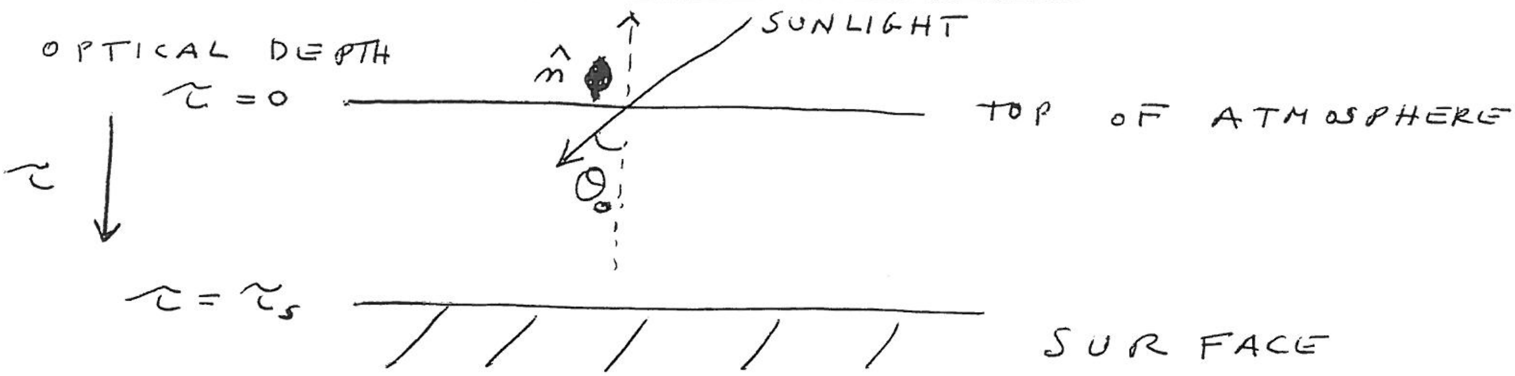
NEED TO SOLVE EQ OF RT DIRECTLY

DIFFICULT BECAUSE

- I IS A FUNCTION OF MORE THAN 1 VARIABLE — τ + DIRECTION
- RT EQ IS INTEGRAL/DIFFERENTIAL EQ.

Need simplifications / numerical

SOLAR PROBLEM



- SUNLIGHT IS UNIDIRECTIONAL:

$$\hat{S} \cdot \hat{n} = -\mu_0 = -\cos\theta_0$$

direction of propogation ($\hat{S}$) upward surface normal ($\hat{n}$)

SOLAR INTENSITY, I_{SUN}, INVOLVES DIRAC DELTA FUNCTION, δ

<u>At $\tau = 0$:</u> $$I_{SUN} = \mathcal{F}_{\odot}\,\delta(\mu + \mu_0)\,\delta(\phi - \phi_0)$$

$\mathcal{F}_{\odot}$: solar FLUX $\perp$ to its direction of propogation

WITHIN ATMOSPHERE, DIRECT SOLAR BEAM

$$I_{SUN}(\tau) = C(\tau)\,\delta(\mu + \mu_0)\,\delta(\phi - \phi_0)$$

EQUATION OF RADIATIVE TRANSFER FOR DIRECT BEAM (UNIDIRECTIONAL):

$$\mu \frac{dI_{SUN}}{d\tau} = I_{SUN}$$

APPLY $\iint d\phi \, d\mu$ to BOTH SIDES OF EQ

$\longrightarrow$

$$-\mu_0 \frac{dC}{d\tau} = C$$

$$\longrightarrow \quad C = C(\tau=0) \exp\left(-\frac{\tau}{\mu_0}\right)$$

$$= \mathcal{F}_\odot \exp\left(-\frac{\tau}{\mu_0}\right)$$

EQUATION OF RADIATIVE TRANSFER FOR THE TOTAL INTENSITY, I_T (DIRECT + DIFFUSE)

$$\mu \frac{dI_T}{d\tau} = I_T - \tilde{\omega}_0 \int_{-1}^{1}\int_0^{2\pi} I_T \, p(\Theta) \frac{d\varphi \, d\mu}{4\pi}$$

LET: $I_T = I_{SUN} + I_{DIF}$

$\longrightarrow$ direct beam at fixed Θ

$$\mu \frac{dI_{SUN}}{d\tau} + \mu \frac{dI_{DIF}}{d\tau} = I_{DIF} + I_{SUN} - \frac{\tilde{\omega}_0}{4\pi} \mathcal{F}_\odot e^{-\tau/\mu_0} P_{SUN}$$
$$- \tilde{\omega}_0 \int_{-1}^{1}\int_0^{2\pi} I_{DIF} \, p \frac{d\varphi \, d\mu}{4\pi}$$

BUT: $\mu \frac{dI_{SUN}}{d\tau} = I_{SUN}$

$$\longrightarrow \boxed{\mu \frac{dI_{DIF}}{d\tau} = I_{DIF} - \frac{\tilde{\omega}_0 F_\odot}{4} \left(e^{-\tau/\mu_0}\right) P_{SUN} - \tilde{\omega}_0 \int_{-1}^{1}\int_0^{2\pi} I_{DIF} \, p \frac{d\varphi \, d\mu}{4\pi}}$$

↑ mult. scattering

BASIC EQ. OF RT FOR SOLAR PROBLEM

WHERE

$$F_{\odot} = \frac{\mathcal{F}_{\odot}}{\pi}$$

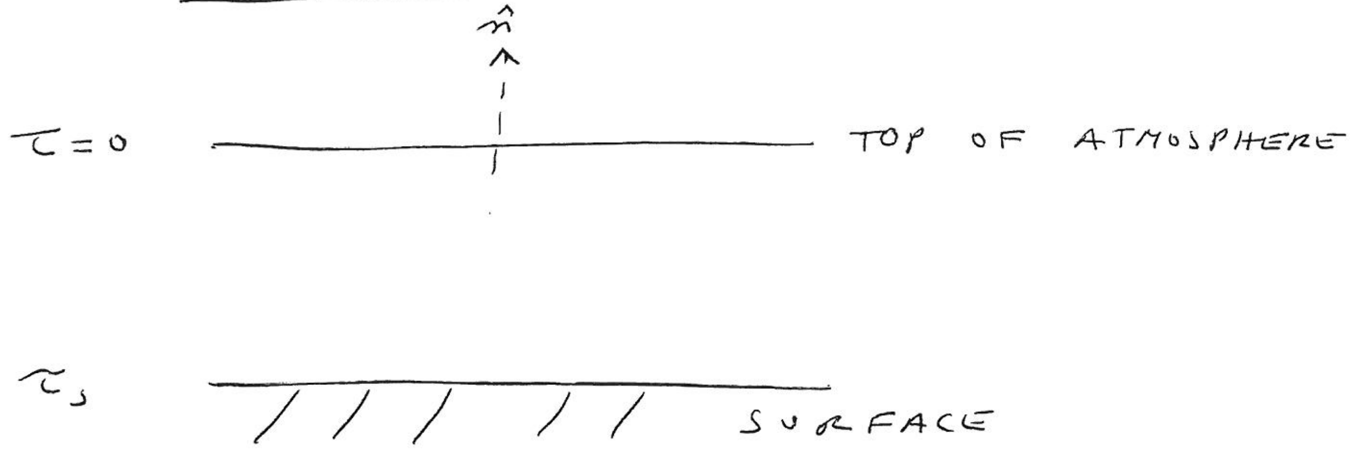
FROM LAMBERT SURFACE PROBLEM

$$P_{SUN} = P(\cos(\Theta'_{\odot}))$$

$$\cos \Theta' = \mu \mu_0 + \sqrt{1-\mu^2}\sqrt{1-\mu_0^2}\cos(\phi - \phi_0)$$

(LAW OF COSINES IN SPHERICAL GEOMETRY)

THERMAL EMISSION PROBLEM



SOURCE OF RADIATION IS THERMAL EMISSION $\longrightarrow$ $S = (1 - \tilde{\omega}_0)\, B$

UNLIKE SOLAR PROBLEM, SOURCE IS:

- OMNIDIRECTIONAL
- ORIGINATES AT ALL LEVELS IN ATMOSPHERE + SURFACE

PURE ABSORPTION PROBLEM $(\tilde{\omega}_0 = 0)$

$$\mu \frac{dI}{d\tau} = I - B$$

$$\longrightarrow \quad \underline{I}^{+} = B_s e^{-\tau_s} + \int_{\tau}^{\tau_s} B e^{-(\tau' - \tau)/\mu} \frac{d\tau'}{\mu}$$

$$\underline{I}^{-} = \int_0^{\tau} B\, e^{-(\tau - \tau')/\mu} \frac{d\tau'}{\mu}$$

(I^{+} IS UPWARD DIRECTED INTENSITY
I^{-} IS DOWNWARD DIRECTED INTENSITY)

2 STREAM APPROXIMATIONS

- WHAT?

  SIMPLIFY THE ANGULAR DEPENDENCE OF THE R.T. EQ → ANALYTICAL SOLUTION.

- OBJECTIVE

  USEFUL FOR ANGULARLY AVERAGED QUANTITIES:

  PLANAR ~~AB~~ ALBEDOS, TRANSMISSIVITIES, FLUX, HEATING RATES, AVERAGE INTENSITY

- HOW?

  MAKE ASSUMPTIONS ON ANGULAR DEPENDENCE OF INTENSITY AND PHASE FUNCTION

SOLUTIONS

2 STREAM

GENERIC EQ RT

The general equation of radiative transfer in a plane parallel scattering atmosphere is

$$\mu \frac{\partial I_v}{\partial \tau_v}(\tau_v, \mu, \phi) = I_v(\tau_v, \mu, \phi) - S_v(\tau_v, \mu, \phi) - \frac{\omega_{0v}}{4\pi} \cdot \int_0^{2\pi} \int_{-1}^{1} P_v(\mu, \mu', \phi, \phi') I_v(\tau_v, \mu', \phi')\, d\mu'\, d\phi' \quad (1)$$

I is DIFFUSE INTENSITY IN CASE OF SOLAR RADIATION

Here μ is the cosine of the angle at which the intensity, I_v, is observed with the angle measured from the outward surface normal, τ is the optical depth measured along the zenith direction beginning at the top of the atmosphere, ω_0 is the single scattering albedo, P is the scattering phase function, and v is the frequency.

For an emitting atmosphere,

$$S_{ve} = (1 - \omega_0) B_v(T) \quad (2)$$

← THERMAL EMISSION

while for a purely external source at solar wavelengths,

$$S_{vs} = \frac{\omega_{0v}}{4} F_{sv} P_v(\mu, -\mu_0, \phi, \phi_0) \exp(-\tau_v/\mu_0) \quad (3)$$

← DIRECT SOLAR

INTEGRATE OVER AZIMUTH ∡ ϕ + ZENITH ∡ μ

(note all RT quantities depend only on $(\phi - \phi_0)$ → $\int d\phi$ removes azimuthal dependance on <u>both</u> ϕ + ϕ_0)

Equation (1) may be integrated over azimuth and zenith angle to yield

$$\frac{\partial F_v^{\pm}}{\partial \tau_v} = \pm \int_0^1 I_v^{\pm}(\tau_v, \mu)\, d\mu \mp \frac{1}{2} \int_0^1 \int_{-1}^{1} P_v(\mu, \mu') \cdot I_v^{\pm}(\tau_v, \mu')\, d\mu\, d\mu' \mp \int_0^{2\pi} \int_0^1 S_v(\tau_v)\, d\mu'\, d\phi' \quad (4)$$

where the diffusive flux in the upward (downward) direction is

$$F_v^{\pm} = \int_0^1 \mu I_v^{\pm}(\tau_v, \mu)\, d\mu \quad (5)$$

The azimuthally integrated intensity is

$$I_v^{\pm}(\tau, \mu) = \int_0^{2\pi} I_v(\tau, \pm\mu, \phi)\, d\phi \quad (6)$$

while

$$\int_0^1 \int_0^{2\pi} S_{ve}(\tau)\, d\phi'\, d\mu' = 2\pi(1-\omega_0)B_v(T) \qquad (7)$$

$$\int_0^1 \int_0^{2\pi} S_{vs}(\tau)\, d\phi'\, d\mu' = \pi F_v \omega_{0v} \beta_{0v} \exp(-\tau_v/\mu_0) \qquad (8)$$

$$\beta_{0v} = \frac{1}{2}\int_0^1 P_v(\mu_0, -\mu')\, d\mu' \qquad (9)$$

with

$$P(\mu, \mu') = \frac{1}{2\pi}\int_0^{2\pi} P(\mu, \mu', \phi, \phi')\, d\phi \qquad (10)$$

NOW INTRODUCE 2 STREAM APPROXIMATIONS BY EXPRESSING ANGULAR INTEGRALS IN TERMS OF F^+, F^-, S^+, AND S^-

EXAMPLES:

very good for thermally emitting atmosphere

Linear dependence on μ

EDDINGTON APPROXIMATION:

$$I = I_0 + I_1 \mu$$

$\overline{\langle \cos\theta \rangle}$

$$P(\mu, \mu') = 1 + 3g\mu\mu'$$

(from Legendre polynomial expansion)

keep 2 terms

QUADRATURE

$$I(\tau, \pm\mu) = I(\tau, \pm\mu_1)\,\delta(\mu - \mu_1)$$

quadrature point

eg gauss $\rightarrow \mu_1 = \frac{1}{\sqrt{3}}$

(but note sometimes used in $\int_0^1 \cdot\, d\mu$, instead of $\int_{-1}^1 d\mu$)

QUADRATURE (CONTINUED)

2 choices:

(1) Use $p = 1 + 3g\mu\mu'$ ~~for~~ in phase function integrals

(2) do numerical integral. (e.g.)

$$\beta_0 = 1 - \frac{1}{2}\int_0^1 p(\mu, \mu_0)\, d\mu$$

(solar flux term)

$$\beta_1 = 1 - \frac{1}{2}\int_0^1 p(\mu, \mu_1)\, d\mu$$

(diffuse scattering term)

(1) is used below, but

(2) is better, especially low τ, low $\tilde{\omega}_0$ (direct solar beam dominant) — however, need to do more pre- ~~comput~~ computations (esp. β_0)

HEMISPHERIC MEAN

$$I(\tau, \pm\mu) = I^{\pm}(\tau) \quad \text{(a constant)}$$

$$p(\pm\mu, \mu') = 1 \pm g$$

USING ANY OF THE ABOVE 2 STREAM APPROXIMATIONS $\longrightarrow$ PAIR OF COUPLED DIFFERENTIAL EQS:

=0 homog. soln (see next page)

Toon paper

$$\frac{\partial F_n^+}{\partial \tau_n} = \gamma_{1n} F_n^+ - \gamma_{2n} F_n^- - \underbrace{S_n^+}_{=0} \qquad (11)$$

$$\frac{\partial F_n^-}{\partial \tau_n} = \gamma_{2n} F_n^+ - \gamma_{1n} F_n^- + \underbrace{S_n^+}_{=0} \qquad (12)$$

Here γ_1 and γ_2 are coefficients which depend upon the particular form of the two-stream equation. Table 1 presents the values of γ_1 and γ_2 for some common two-stream approximations. We anticipate a multiple-layer solution by labeling the flux by n, the layer number.

TABLE 1. Summary of Coefficients in Selected Two-Stream Approximations

Method*	γ_1	γ_2	γ_3†	μ_1
Eddington	$[7 - \omega_0(4 + 3g)]/4$	$-[1 - \omega_0(4 - 3g)]/4$	$(2 - 3g\mu_0)/4$	1/2
Quadrature	$(3)^{1/2}[2 - \omega_0(1 + g)]/2$	$\omega_0(3)^{1/2}(1 - g)/2$	$[1 - (3)^{1/2} g\mu_0]/2$‡	$1/(3)^{1/2}$
Hemispheric mean	$2 - \omega_0(1 + g)$	$\omega_0(1 - g)$	...	1/2

*The Eddington and quadrature schemes are discussed in detail by *Meador and Weaver* [1980]. The hemispheric mean scheme is derived by assuming that the phase function is equal to $1 + g$ in the forward scattering hemisphere and to $1 - g$ in the backward scattering hemisphere. The asymmetry parameter is g.

†$\gamma_4 = 1 - \gamma_3$.

‡Only needed for solar wavelengths. However, the hemispheric mean is only useful for infrared wavelengths.

For the solar beam,

$$S^+ = \gamma_3 \pi F_s \omega_0 \exp\left[-(\tau_c + \tau)/\mu_0\right] \qquad (13)$$

$$S^- = \gamma_4 \pi F_s \omega_0 \exp\left[-(\tau_c + \tau)/\mu_0\right] \qquad (14)$$

where γ_3 and γ_4 are coefficients which depend on the two-stream equations used [*Meador and Weaver*, 1980]. Some examples are given in Table 1. When multiple layers are used, τ_c is the cumulative optical depth of layers above layer n.

Equations (11) and (12) admit to simple solutions for any function S that can be written as any linear combination of τ, exponentials in τ, or sines and cosines of τ. The Planck function can often be approximated in such a fashion. Then one can solve (11) and (12) for the infrared with

$$S^+ = S^- = 2\pi(1 - \omega_0)B(\tau) \qquad (15)$$

where B is the Planck function expressed in terms of the layer optical depth.

ASSUME THAT $B(\tau) = B_{0n} + B_{1n}\tau$

(SO NEED TO DIVIDE ATMOSPHERE INTO SERIES OF LAYERS) particular solutions

solutions of homogeneous eqns

The general solution to the two-stream equations (11) and (12) can be shown to be

$$F_n^+(\tau)=k_{1n} \exp(\lambda_n\tau) + \Gamma_n k_{2n} \exp(-\lambda_n\tau)+C_n^+(\tau) \qquad (19)$$

$$F_n^-(\tau)=\Gamma_n k_{1n} \exp(\lambda_n\tau)+k_{2n} \exp(-\lambda_n\tau)+C_n^-(\tau) \qquad (20)$$

These expressions, which again have a layer index n, apply for all cases except those in which ω_0 is exactly unity. In this case the homogeneous part of the solutions reduces to linear functions of τ. It can be shown that as ω_0 approaches unity, then (19) and (20), when coupled with the boundary conditions, approach these linear solutions.

The terms k_1 and k_2 in (19) and (20) are determined by boundary conditions, while λ and Γ depend upon the form of the two-stream equation used.

$$\lambda = (\gamma_1^2 - \gamma_2^2)^{1/2} \qquad (21)$$

$$\Gamma = \frac{\gamma_2}{(\gamma_1+\lambda)} = \frac{\gamma_1-\lambda}{\gamma_2} \qquad (22)$$

EXAMPLE OF BOUNDARY CONDITIONS

ONE LAYER: ~~[illegible]~~ down sun in S

$I^-(\tau=0) = 0$ (NO DIFFUSE RADIATION AT TOP OF ATMOSPHERE

$I^+(\tau_s) = \lambda I^-(\tau_s)$ solar, lambert surface

or $= e_s B(T_s)$, thermal emission

WITH MULITIPLE LAYERS, REQUIRE CONTINUITY OF $I^\pm$ AT INTERIOR BOUNDARIES

SOLAR

For solar radiation,

$$C^{+}(\tau)=\frac{\omega_0\pi F_s\exp\left[-(\tau_c+\tau)/\mu_0\right]\left[(\gamma_1-1/\mu_0)\gamma_3+\gamma_4\gamma_2\right]}{(\lambda^2-1/\mu_0^2)} \tag{23}$$

$$C^{-}(\tau)=\frac{\omega_0\pi F_s\exp\left[-(\tau_c+\tau)/\mu_0\right]\left[(\gamma_1+1/\mu_0)\gamma_4+\gamma_2\gamma_3\right]}{(\lambda^2-1/\mu_0^2)} \tag{24}$$

THERMAL

Using this approximation of the Planck function,

LINEAR IN τ
COEFFICIENTS
FOUND FROM
B(T) AT LAYER
BOUNDARIES

$$\begin{aligned} C_n^{+}(\tau) &= 2\pi\mu_1\{B_{0n}+B_{1n}[\tau+1/(\gamma_{1n}+\gamma_{2n})]\} \\ C_n^{-}(\tau) &= 2\pi\mu_1\{B_{0n}+B_{1n}[\tau-1/(\gamma_{1n}+\gamma_{2n})]\} \end{aligned} \tag{27}$$

ADDITIONAL SOPHISTICATIONS

DELTA APPROXIMATION!

MOTIVATION

PHASE FUNCTIONS USUALLY HAVE
STRONG FORWARD PEAK

SMALL Θ IN PEAK → ALMOST LIKE
UNSCATTERED LIGHT

Joseph paper

APPROACH

REPLACE ACTUAL PHASE FUNCTION
WITH SUM OF δ FUNCTION IN
FORWARD DIRECTION + LESS SHARPLY
PEAKED REMAINER

→ MORE ACCURACY IN
2 STREAM APPROX.

$$P_\delta(\cos \Theta) \cong 2 f \delta(1 - \cos(\Theta)) + (1-f)\underbrace{(1 + 3g' \cos \Theta)}_{\text{Legendre polynomial expansion – 1st 2 Term}}$$

f = fraction of scattering in forward peak

Require $\langle \cos \Theta \rangle$ for P_δ =
g = $\langle \cos \Theta \rangle$ of real phase function

$$\longrightarrow \quad g' = \frac{(g - f)}{(1 - f)}$$

Require 2nd moment of P_δ equal that of real phase function, which is approximated by a Henyey-Greenstein phase function $\longrightarrow$

$$f = g^2$$

& therefore

$$g' = \frac{g}{1+g}$$

Also need to scale the single scattering albedo & optical depth for loss of forward peak $\longrightarrow$

$$\tau' = (1 - \tilde{\omega} f)\tau$$

$$\tilde{\omega}' = \frac{(1-f)\tilde{\omega}}{1 - \tilde{\omega} f}$$

(' Means δ Values)

2 STREAM SOURCE FUNCTION
(FOR THERMAL RADIATION)

PURPOSE

KEEP SOME EMISSION & INFORMATION

APPROACH

USE 2 STREAM $\longrightarrow$ SOURCE FUNCTION

USE FORMAL SOLUTION OF RT EQ

$\longrightarrow I(\tau, \mu)$

ACCURACY AND PHYSICAL REALITY OF 2 STREAM

- USUALLY ACCURATE TO WITHIN 10%
  (HEMISPHERIC MEAN)
- 2 STREAM SOURCE FUNCTION BEST FOR THERMAL EMISSION
- δ EDDINGTON OR δ QUADRATURE BEST FOR SOLAR

  but

  BETTER NOT TO APPROXIMATE PHASE FUNCTION INTEGRALS
  (IN A FEW CASES → NEGATIVE ALBEDOS – WHEN $\gamma_3 < 0$)

TABLE 2. Limiting Cases for Various Approaches

Case	δ-Quadrature	δ-Eddington	δ-Hemispheric Mean	Source Function
		Solar		
$\omega_0 = 1$, flux conserved	yes	yes	yes	...*
$\omega_0 = 0$, flux down exact	yes	maybe†	yes	...
		Infrared		
$\omega_0 = 0$, fluxes exact	no	no	no	yes
$\omega_0 = 0$, $\tau = \infty$, emissivity = 1	no	no	yes	yes
$\omega_0 = 1$, flux conserved	yes	yes	yes	yes

*The source function technique is not considered here at solar wavelengths.
†Maybe is exact if no reflecting surfaces are present.

- USE OF MULITIPLE LAYERS DOES NOT ALTER LEVEL OF ERRORS

No τ in error

δ QUADRATURE COMPARISONS

TABLE 4. Errors in Diffusively Transmitted and Reflected Boundary Fluxes

	Case Number				
	1	2	3	4	5
		Parameters for Case			
μ_0	1	1	0.5	1	1
τ	1	1	1	64	64
ω_0	1	0.9	0.9	1	0.9
g	0.794	0.794	0.794	0.848	0.848
		Results for Case			
$F^+(0)$ exact	0.173	0.124	0.226	2.662	0.376
$F^+(0)$ approximate	0.174	0.133	0.221	2.686	0.376
% error (F^+)	0.6	7	2	0.9	0
$F^-(\tau)$ exact	1.813	1.516	0.803	0.480	0.0000
$F^-(\tau)$ approximate	1.812	1.522	0.864	0.455	0.000
% error (F^-)	0	0.3	8	5	0

In all of the cases, $R_{sfc} = 0$, and $F_s = 1$ so that the incident solar flux equals π. Exact values are from *Wiscombe* [1977] or *Lenoble* [1985].

TABLE 5. Errors in Net Flux and Flux Divergence for Multiple Layers

	Case 1		Case 2		Case 3		Case 4		Case 5	
τ Level	Flux	Divergence	Flux	Divergence	Flux	Divergence	Flux	Divergence	Flux	Divergence
$\tau = 0$	2.968	0	3.018	0.0017	1.345	0.019	0.480	0	2.776	1.161
$\tau = 0$	2.967	0	3.009	0.0017	1.350	0.018	0.455	0	2.766	1.161
$\tau = 0$	(0.03)	(0)	(0.3)	(0)	(0.3)	(5)	(5)	(0)	(0)	(0)
$\tau/20$	2.968	0	3.001	0.0018	1.326	0.020	0.480	0	1.605	0.775
$\tau/20$	2.967	0	2.992	0.0017	1.332	0.017	0.455	0	1.605	0.783
$\tau/20$	(0.03)	(0)	(0.3)	(5)	(0.5)	(15)	(5)	(0)	(0)	(1)
$\tau/10$	2.968	0	2.983	0.0035	1.306	0.040	0.480	0	0.830	0.634
$\tau/10$	2.967	0	2.975	0.0034	1.315	0.032	0.455	0	0.822	0.639
$\tau/10$	(0.03)	(0)	(0.3)	(3)	(0.7)	(20)	(5)	(0)	(1)	(1)
$\tau/5$	2.968	0	2.948	0.1060	1.266	0.109	0.480	0	0.196	0.194
$\tau/5$	2.967	0	2.941	0.1010	1.283	0.088	0.455	0	0.183	0.182
$\tau/5$	(0.03)	(0)	(0.2)	(4)	(1)	(19)	(5)	(0)	(7)	(6)
$\tau/2$	2.968	0	2.842	0.0870	1.157	0.078	0.480	0	0.0022	0.0021
$\tau/2$	2.967	0	2.840	0.0820	1.195	0.064	0.455	0	0.0013	0.0013
$\tau/2$	(0.03)	(0)	(0.1)	(6)	(1)	(13)	(5)	(0)	(40)	(39)
$3\tau/4$	2.968	0	2.755	0.0840	1.079	0.063	0.480	0	0.0001	0.0001
$3\tau/4$	2.967	0	2.758	0.0810	1.131	0.054	0.455	0	0.00002	0.00002
$3\tau/4$	(0.03)	(0)	(0.1)	(4)	(5)	(14)	(5)	(0)	(80)	(80)
τ	2.968		2.671		1.016		0.480		0.0000	
τ	2.967		2.677		1.077		0.455		0.0000	
τ	(0.03)		(0.2)		(6)		(5)		(0)	

First value is exact, second value is δ-quadrature result, and percent error is in parentheses. Net flux is at optical depth indicated. Divergence is for layer between adjacent optical depth levels. Cases and τ are defined in Table 4.

THERMAL EMISSIVITY COMPARISONS

E = exact answer

SF3,8 = error in source function w/ 3, 8 quadrature points

δE = error in δ EDDINGTON

δH = error in hemispheric mean

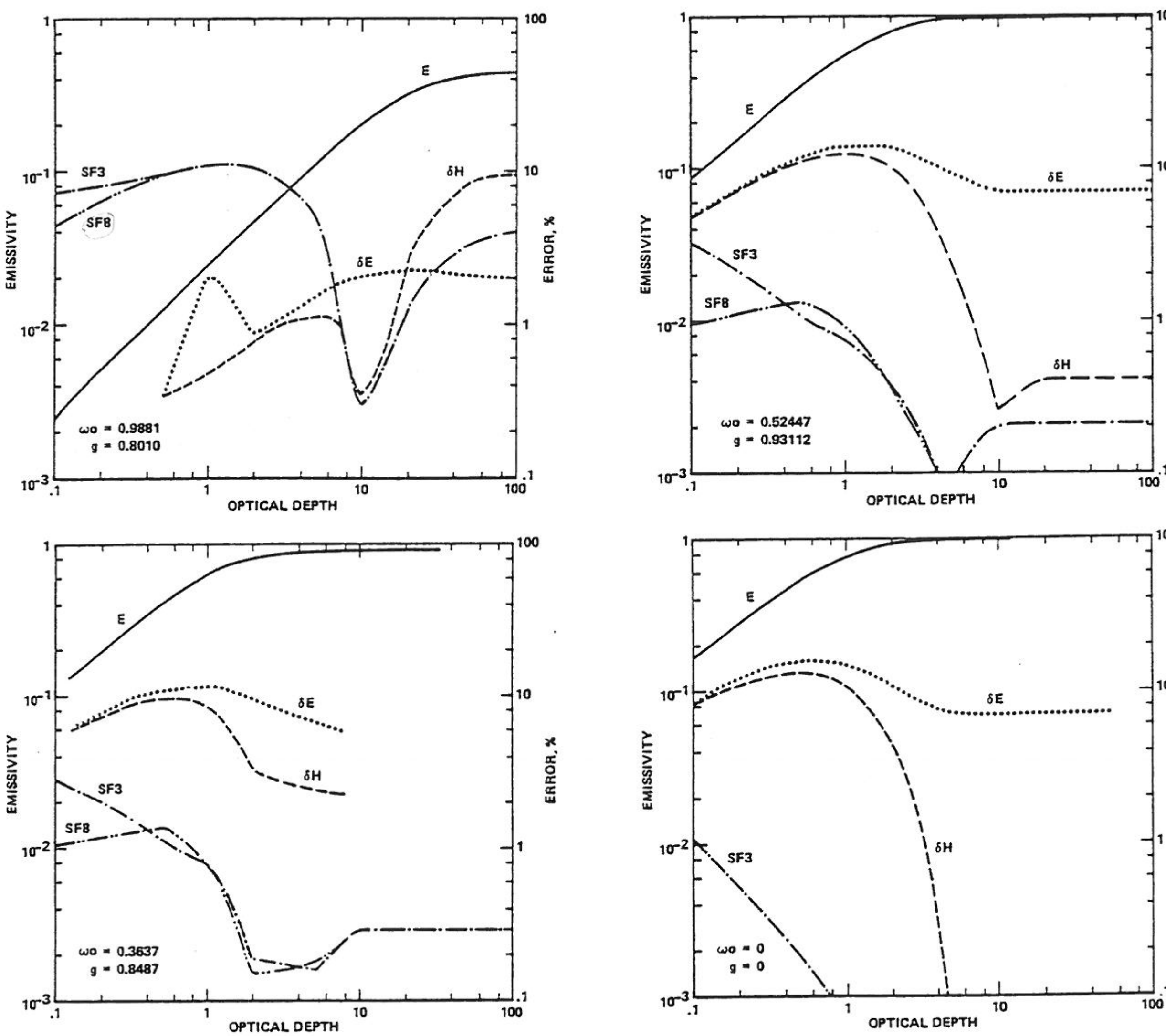


Fig. 1. Shown in each segment of the figure are the emissivity (*E*) for an isothermal multiple-scattering layer as calculated precisely [*Hunt*, 1973] as well as the errors in calculating the emissivity with the δ-hemispheric mean technique (δH), the δ-Eddington technique (δE), and the source function technique in which the δ-hemispheric mean approach is used to obtain the two-stream approximation to the source function. For the source function technique, results are shown for Gauss quadratures using both three (SF3) and eight (SF8) Gauss points to obtain the fluxes. The left-hand scale applies to the emissivity and the right-hand scale to the error (in percent). The various panels cover a range of values for the asymmetry parameter and the single scattering albedo. The sign of the error is a function of both the technique and the optical depth. In order to emphasize the magnitude of the error in a clear manner, only the absolute value of the error is shown.

NOTE – source function does very good as $\hat{\omega}_0 \rightarrow 0$, as expected

2 STREAM APPLICATION

NIGHTSIDE ~~[illegible]~~ NEAR-IR EMISSION FROM VENUS

WHAT?

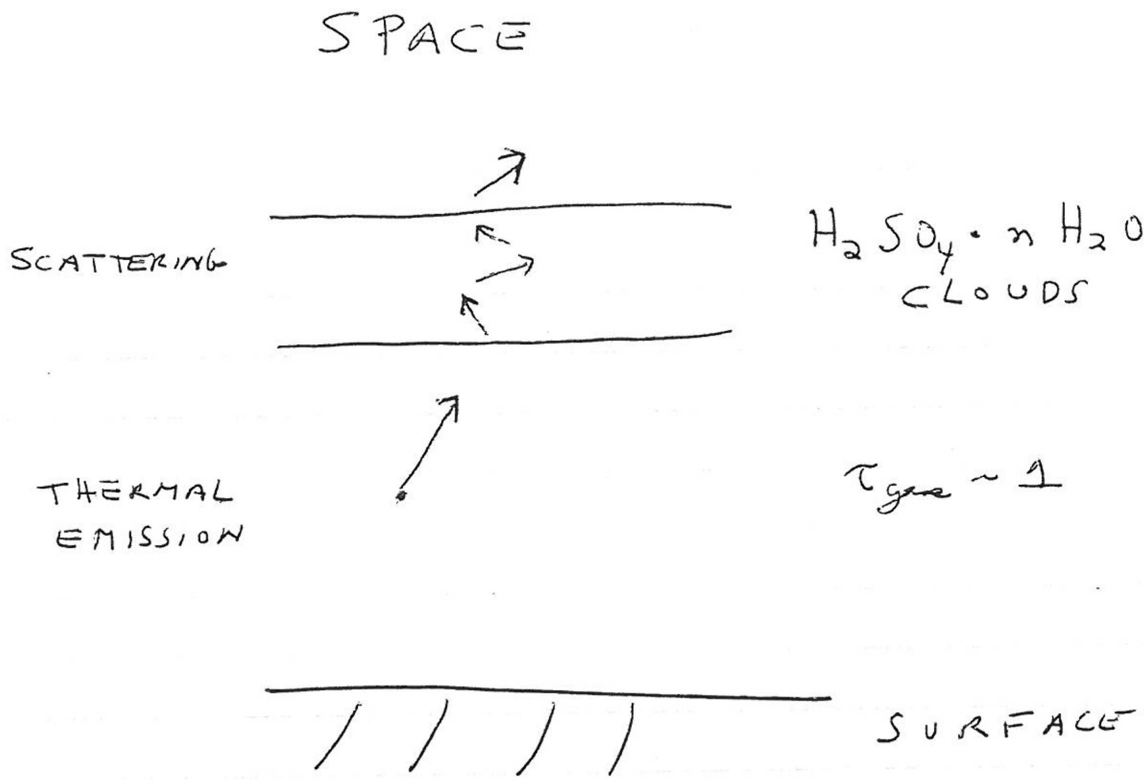
- OBSERVE EMISSION IN SELECTED WAVELENGTH REGIONS OF $1-2.5\,\mu m$ DOMAIN FROM NIGHTSIDE
  ($\neq$ SCATTERED LIGHT FROM DAYSIDE)
- EMISSION SHOWS STRONG SPATIAL VARIABILITY

SOURCE OF EMISSION:

- THERMAL RADIATION FROM HOT LOWER ATMOSPHERE BENEATH MAIN CLOUDS
- CLOUDS SCATTER & ABSORB THERMAL RADIATION FROM BELOW, BUT TOO COLD AT $\lambda < 2.5\,\mu m$ TO EMIT

CONDITIONS FOR WINDOW

- GAS ABSORPTION WEAK $\rightarrow \tau = 1$ DEEP
  $\rightarrow$ SIGNIFICANT THERMAL EMISSION;
  ON WIEN TAIL OF BLACKBODY FUNCTION $\sim \exp\left(-\frac{c}{T}\right)$
- CLOUD PARTICLES HAVE VERY HIGH SINGLE SCATTERING ALBEDO, $\tilde{\omega}_0 \sim 1$, $\rightarrow$ SIGNIFICANT TRANSMISSION EVEN THOUGH $\tau_{cloud} \approx 30$



IF $\tilde{\omega}_0 = 0$, Transmission, Tr

$$Tr \approx \exp(-\tau_{cloud}) \lll 1$$

IF $\tilde{\omega}_0 = 1$,

$$Tr \approx \frac{1}{1 + (1-\underset{\langle \cos\theta \rangle}{g})\,\tau_{cloud}}$$

— SPATIAL VARIATION OF EMISSION DUE TO VARIATION IN CLOUD OPACITY

EXAMPLE OF SPECTRUM FOR 2.3 μm WINDOW

LINE STRUCTURE, $\lambda > 2.3 \mu m$, $\rightarrow$ INFO ON GAS ABUNDANCES BELOW CLOUDS

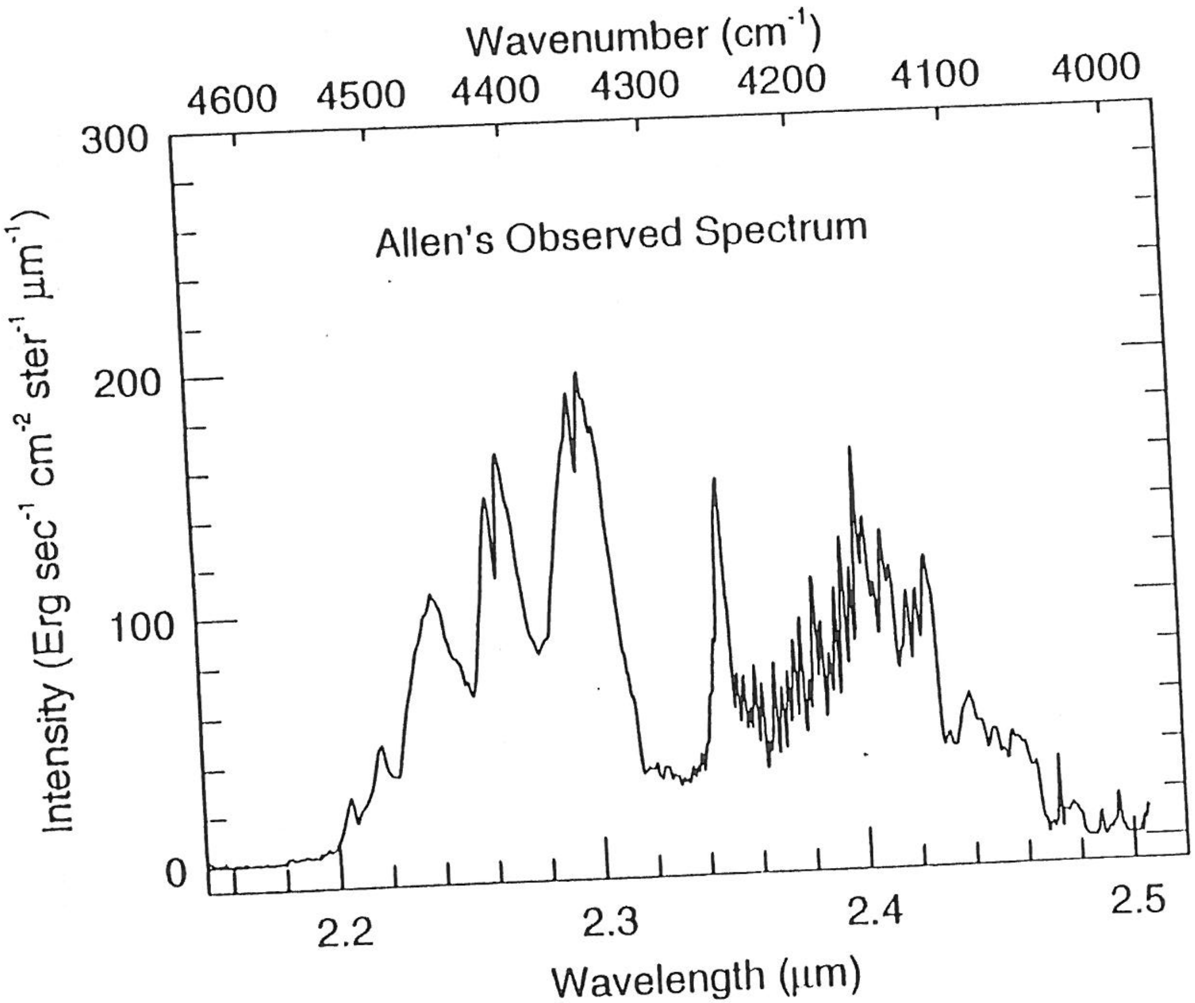


Fig 4a.

APPROACH TO MODELING

- GAS DATABASE
  - NEED TO INCLUDE HOTBANDS, HIGH J ROTATIONAL LINES $\rightarrow$
    $\neq$ HITRAN (ESP. CO_2), USE HIGH T
  - NEED TO CONSIDER CONTINUUM OPACITY
    - FAR WINGS STRONG CO_2 LINES
    - CO_2 PRESSURE INDUCED OPACITY
    - H_2O CONTINUUM
    - DATABASES INCOMPLETE, INACCURATE $\rightarrow$ ADJUSTABLE CONSTANT CONTINUUM IN A GIVEN WINDOW

- CLOUD PROPERTIES
  - USE PIONEER VENUS SIZE DISTRIBUTION (TRI-MODAL)
  - OPTICAL CONSTANTS OF CONCENTRATED SULFURIC ACID
  - ADJUST τ_{cloud} to GET OBSERVED ABSOLUTE VALUES OF THE INTENSITY IN A GIVEN WINDOW
  - USE MIE THEORY (PARTICLES ARE LIQUID $\rightarrow$ SPHERES)

APPROACH (CONTINUED)

- CHOICE OF RT PROGRAM
  - USE HEMISPHERIC MEAN, 2 STREAM SOURCE FUNCTION SOLUTION
  - HIGH VERTICAL RESOLUTION BELOW CLOUDS (≈ 2 km) SO LINEARIZATION OF PLANCK FUNCTION OK ($B \simeq B_0 + B_1 \tau$)
  - VARY GAS MIXING RATIOS TO MATCH STRUCTURE IN OBSERVED SPECTRUM
  - USE PIONEER VENUS TEMPERATURE STRUCTURE

COMPARISON OF CO_2 OPACITIES
DERIVED FROM HITRAN 91
AND FROM WATTSON'S HIGH T DATABASE
FOR 2.3 μm WINDOW →
USE HIGH T CO_2 DATABASE

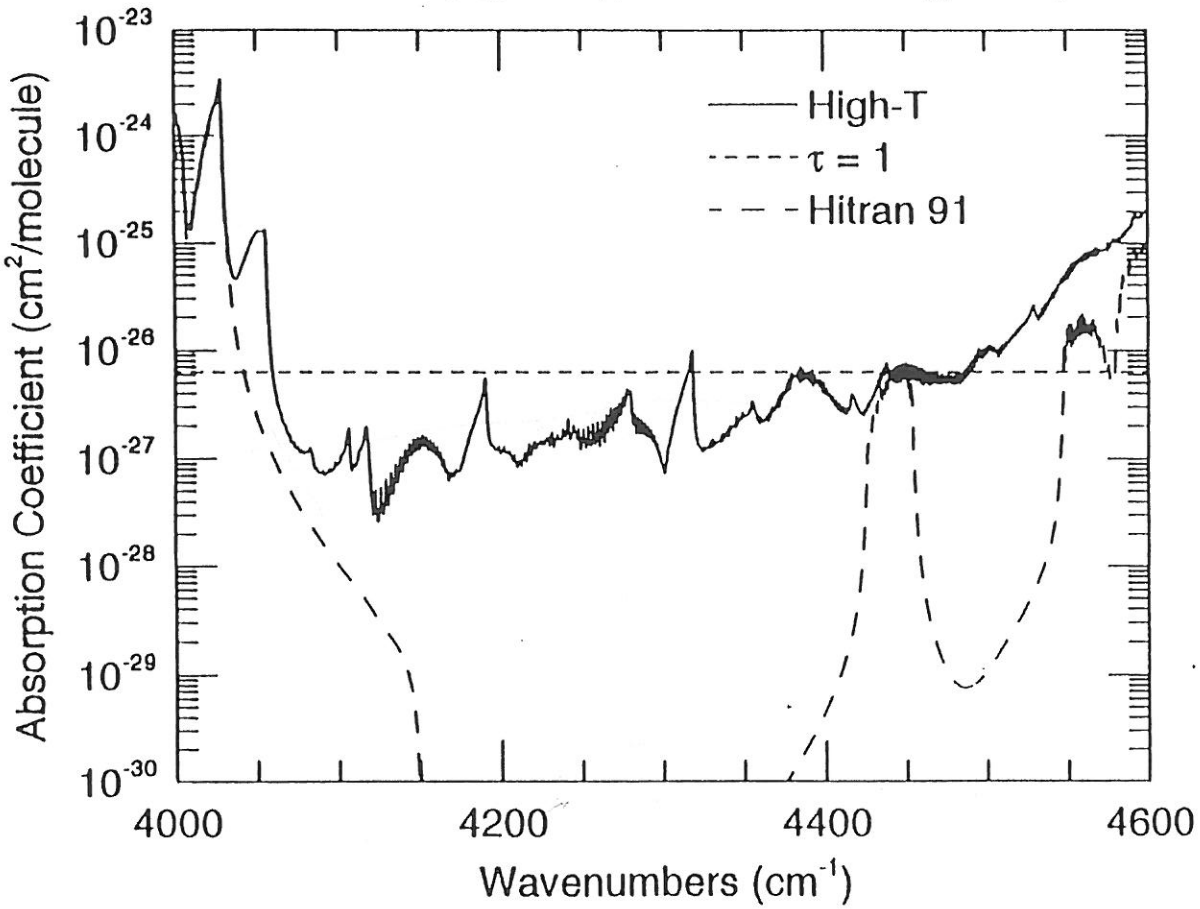
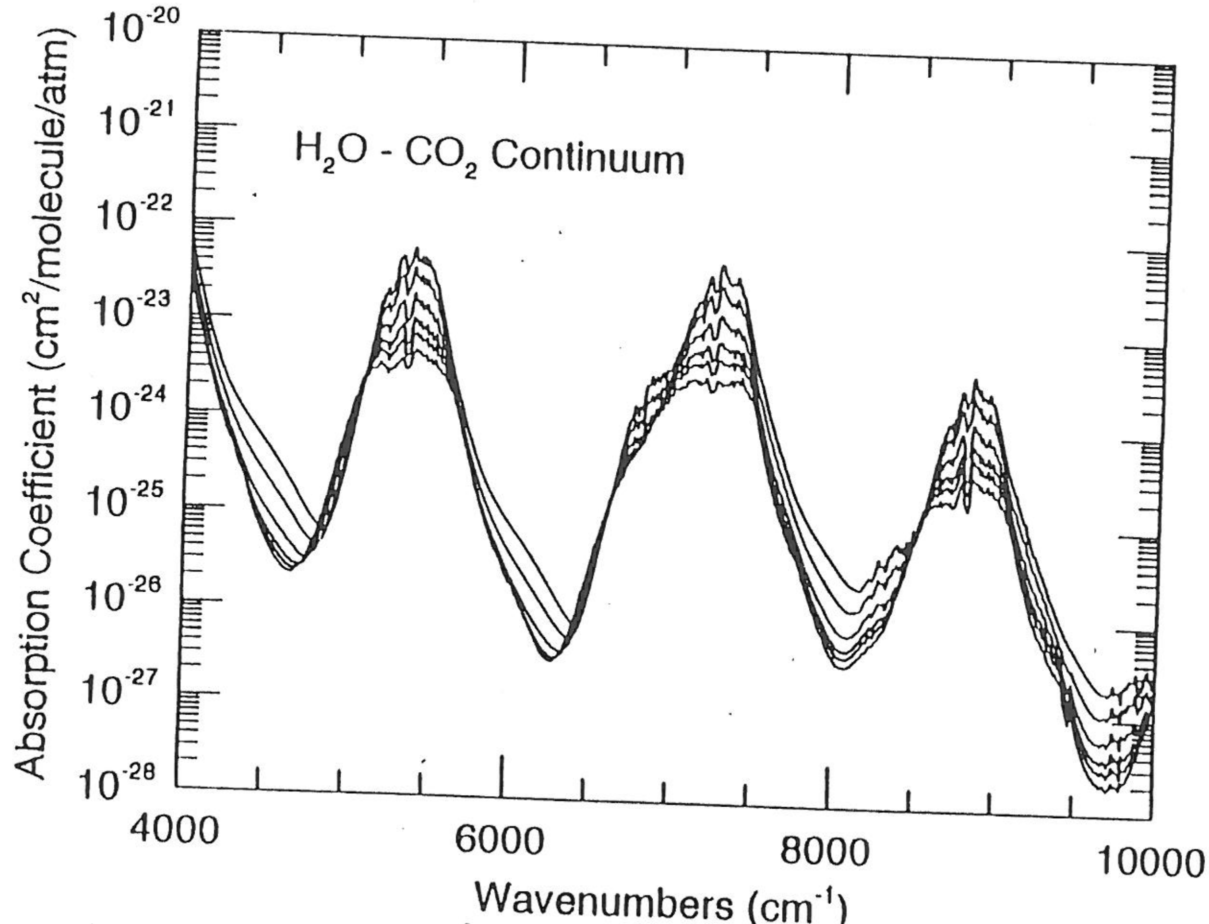


Fig. 1a



H_2O - CO_2 Continuum
Absorption Coefficient (cm^2/molecule/atm)
10^{-20}
10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^{-25}
10^{-26}
10^{-27}
10^{-28}
4000
6000
8000
10000
Wavenumbers (cm^{-1})

Fig. 3b.

ig. 8

LOCATION OF WINDOWS

BRIGHTNESS TEMPERATURES PROVIDE AN ESTIMATE OF LOCATION OF $\tau_{gas} = 1$

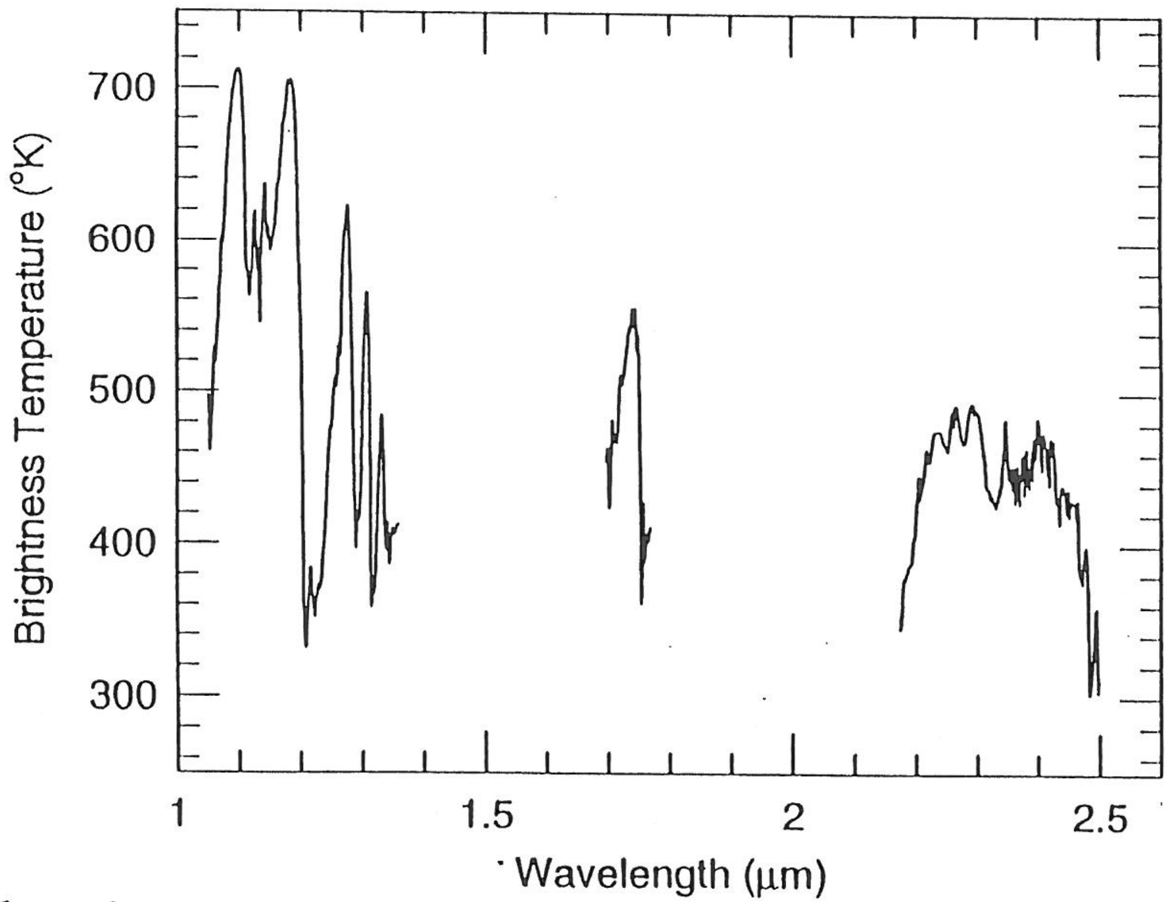


Fig. 8

LOCATION OF WINDOW REGIONS (—)
DETERMINED FROM λ's WHERE
JOINT CO_2/H_2O OPACITY LOW

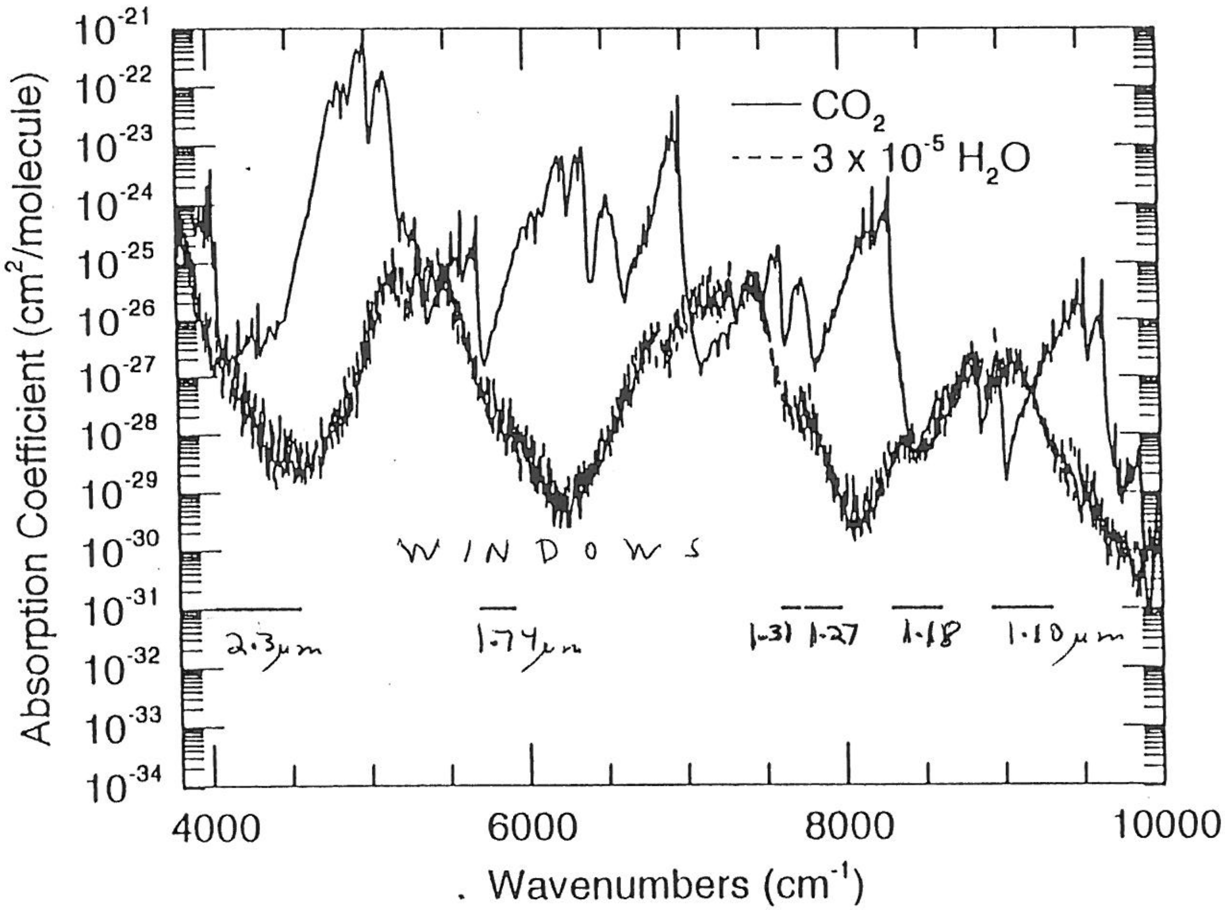


Fig. 7.

DETERMINE RANGE OF ALTITUDES SENSED BY VARYING, ONE AT A TIME, GAS MIXING RATIO OF EACH MODEL LAYER AND FINDING INTENSITY AT TOP OF ATMOSPHERE

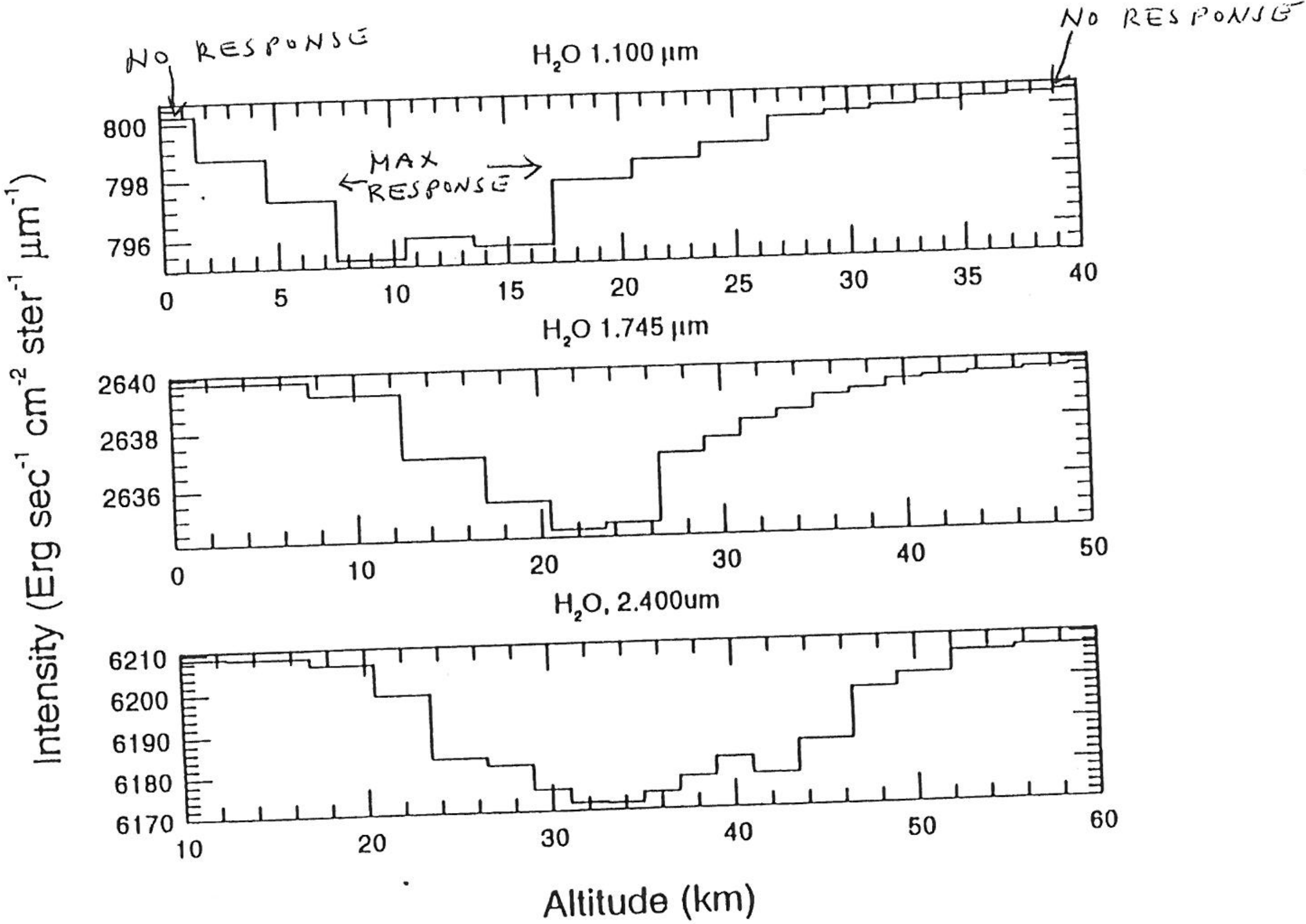


Fig. 9a

USE HIGH RESOLUTION SPECTRUM
OF CO_2 DOMINATED REGION (KNOW CO_2 MIXING RATIO)
TO DERIVE CONSTANT CONTINUUM
IN 2.3μm REGION

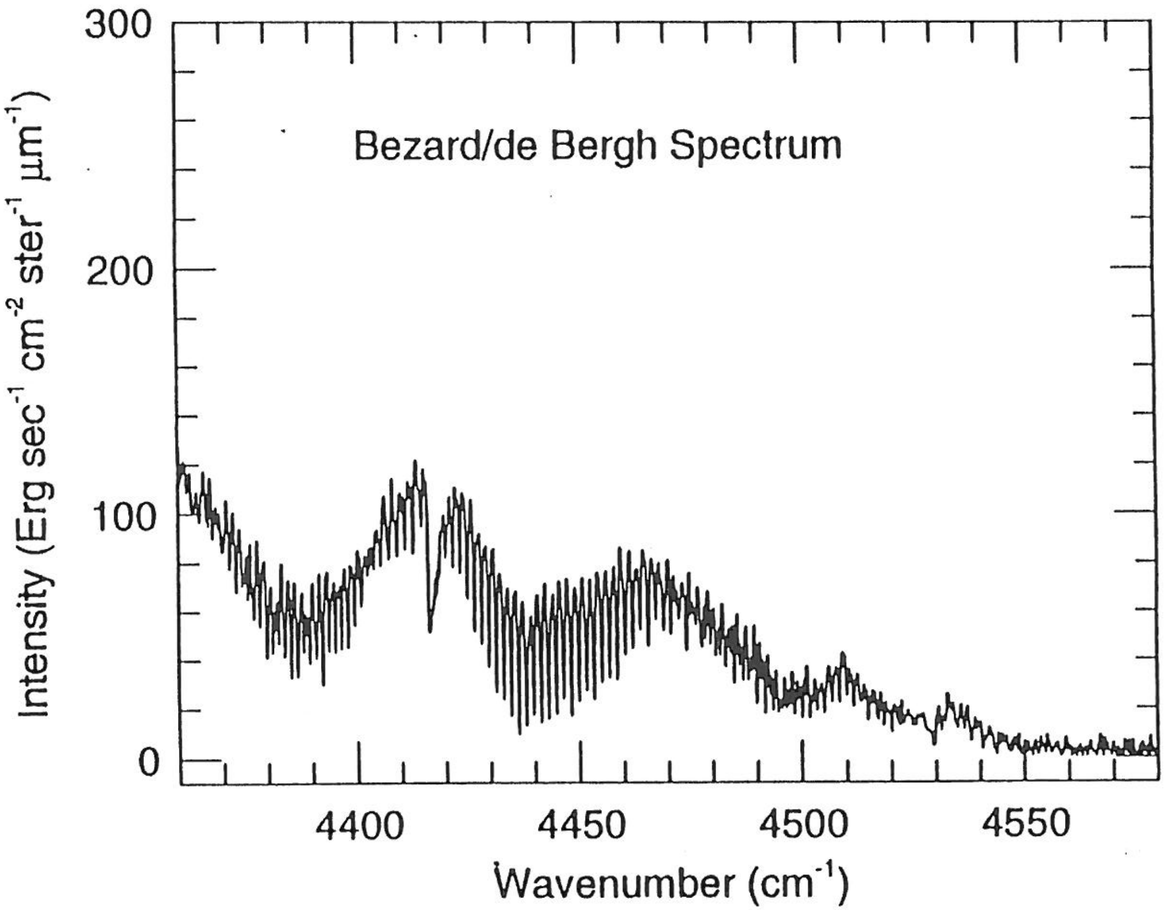


Fig. 10 a.

NOTE CHANGE IN LINE DEPTH
WITH VARYING CONTINUUM COEFFICIENT

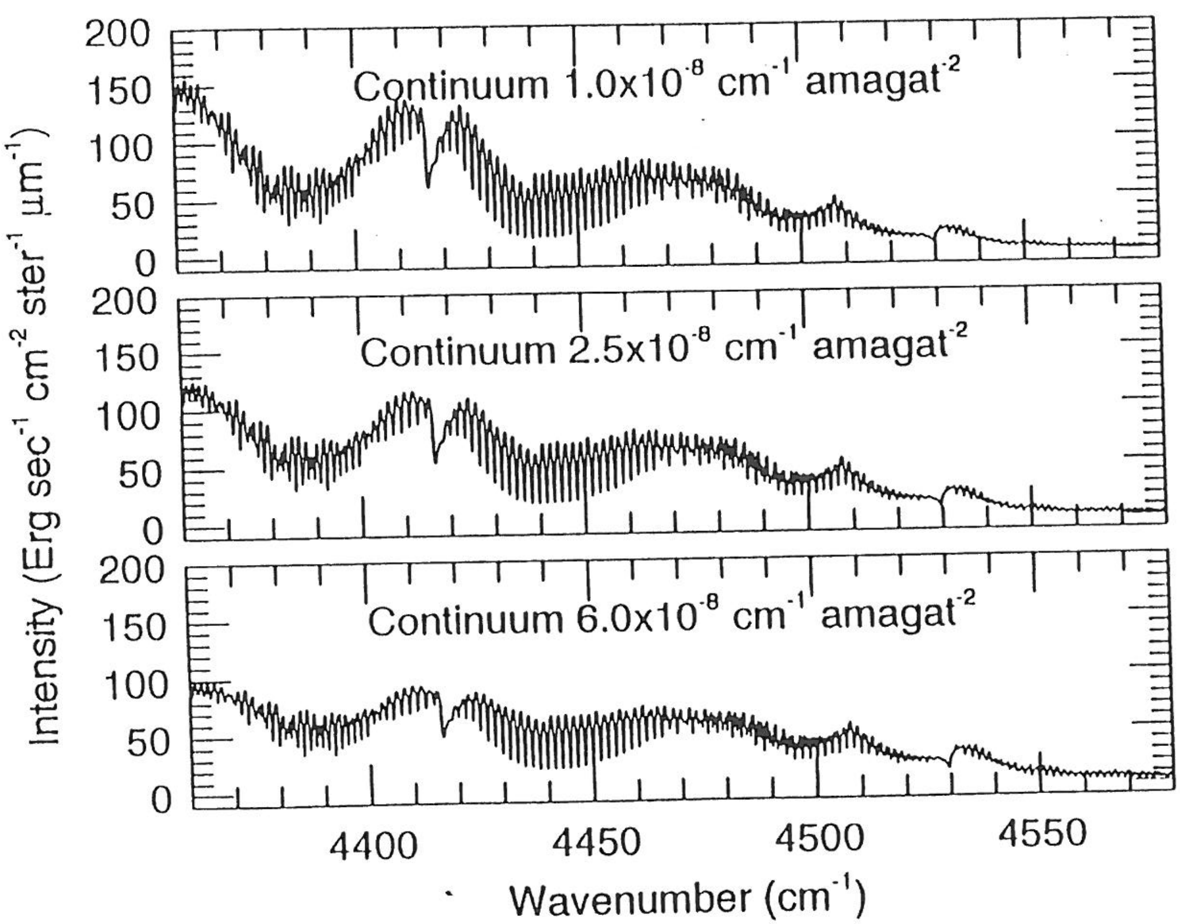


Fig. 10b.

PROCEDURE TO DEFINE WHAT λ's ARE SENSITIVE TO A GIVEN GAS SPECIES

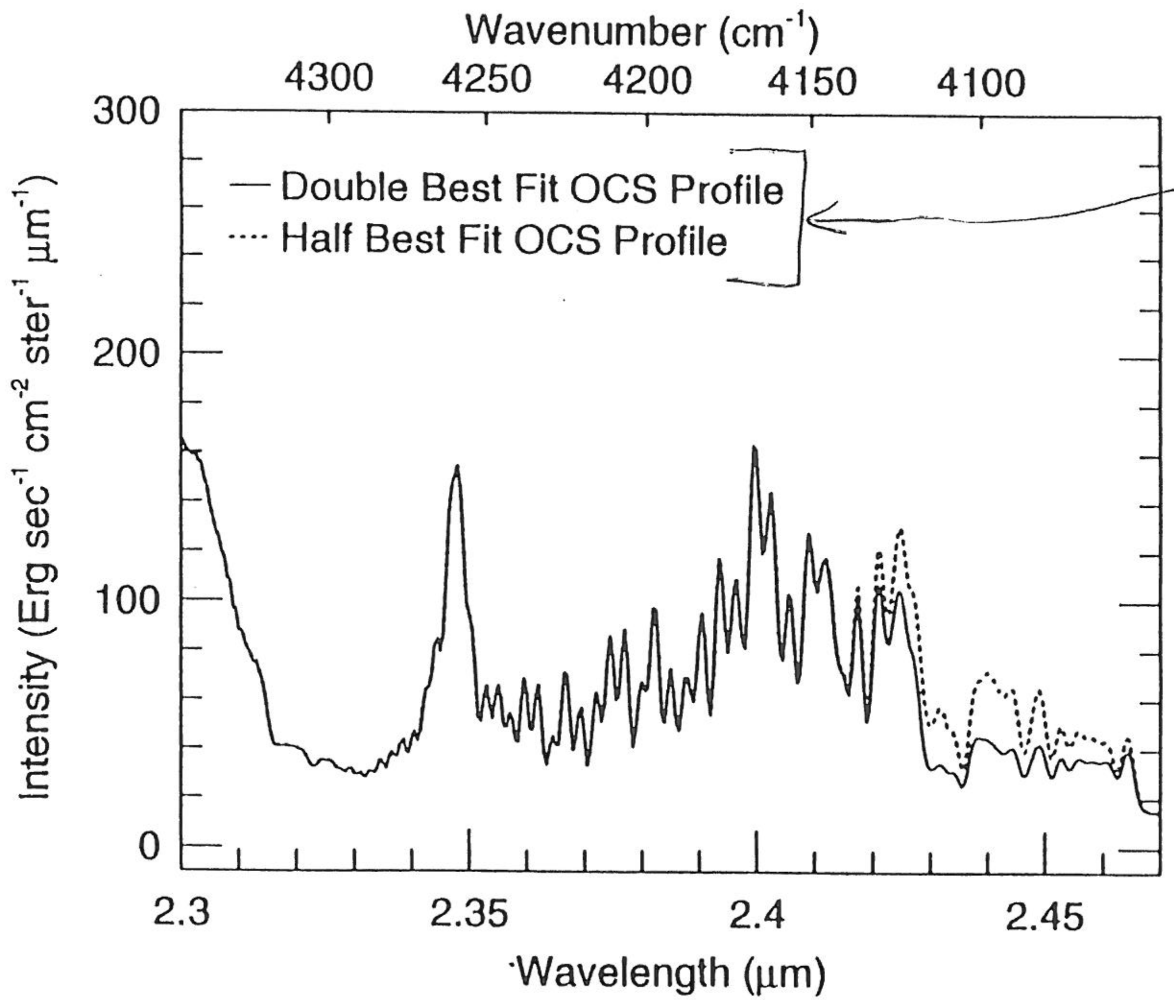


Fig. 12a

NEED ~~F~~ FOR $\frac{d\alpha}{dz} \neq 0$ FOR COS

→ OBTAIN BOTH α & $\frac{d\alpha}{dz}$ FOR SOME GASES AT CENTROID OF EMISSION (BECAUSE KNOW T(P) VERY ACCURATELY)

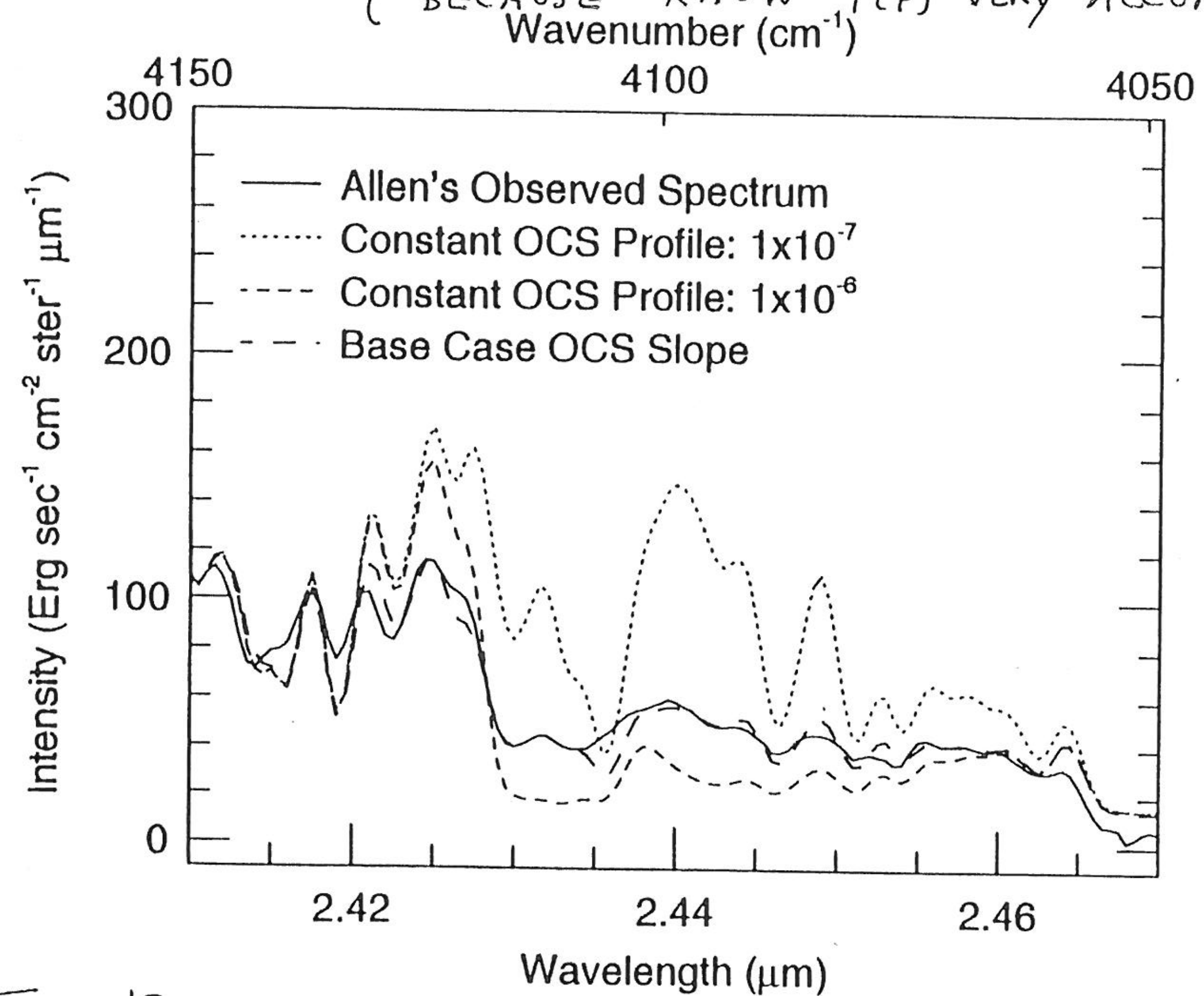


Fig. 13a

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